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Theoretical Simulation of Free Atmospheric Planetary Waves on an Equatorial Beta-plane

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Abstract : A simple theoretical model was developed to investigate behavior of equatorial planetary waves (EPW) in the Earth's atmosphere. Based on linearized equations of EPW in equatorial β-plane, EPW by mode n=0 has greatest meridional wind amplitude at the equator and decays exponentially when away from the equator (Gaussian decay). EPW by mode n=1 has greatest amplitude of meridional wind perturbation (v') at latitude y= $\pm$ 1 (nondim) and EPW by mode n=2 has v' equal to zero at latitude y = $1/2\sqrt{2}$ (nondim), the peak of amplitude is just outside the equator. Simulation of Yanai wave results that both zonal wind and geopotential field have greatest perturbation amplitude at latitude y= $\pm$ 1($u' = 0$ in Equator) while perturbation of zonal wind (u') and geopotential field (Φ') will be in geostrophic balance at latitude $-1 < y < 1$ (nondim) or -$(\beta^{-1}\sqrt{gH})^{1/2} > y > (\beta^{-1}\sqrt{gH})^{1/2}$ (length dimension). Simulation of Kelvin wave results that either zonal wind or geopotential field has symmetric amplitude and symmetric perturbation relative to Earth's latitude. For special treatments, by mode n=1, 2, and 3 there are two classes of EPW, namely high frequency Poincaré modes waves and the low frequency Rossby modes waves. These waves have special behaviors as functions of n. This paper presents a linear model of EPW and mathematically goes step-by-step to derive and explain their character on an equatorial beta-plane.

Keywords : Equatorial Planetary Waves, Kelvin waves, Yanai waves, Rossby waves, Poincaré waves.

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1 INTRODUCTION

Theory of atmospheric waves along the equatorial band is unique, and considered to be one of the driving atmospheric physical phenomena in the tropics. Various studies have been conducted to identify and analyze the behavior of equatorial planetary waves (EPW) which are observed from troposphere to lower stratosphere, such as Kelvin waves, Rossby waves, Yanai waves and Poincaré waves as well [3, 6, 8, 9, 11, 12, 15, 17]. Equatorial waves are generated by diabatic heating due to organized tropical large-scale convective heating in the equatorial belt [6, 9]. Although they do not contain as much energy as other typical tropospheric weather disturbances, EPW causes predominant disturbances in the equatorial atmosphere such as inducing mean-meridional circulation which plays an important role in the heat balance of the equatorial belt, relating to the coldness process of the equatorial tropopause and also affecting the patterns of low level moisture convergence that control the distribution of convective heating and convective storms in large longitudinal distances [1, 6, 9, 10]. Regarding the above information, study of these waves is substantially important to be developed. One of the ways is to simulate these waves within the linear shallow water model on an equatorial β-plane.

Simulation of EPW helps us to explain how these waves can be observed through relation of atmospheric perturbation elements. Matsuno [17] developed a model of quasi-geostrophic motions in the equatorial area by applying a single layer of homogeneous incompressible fluid with free surface and assuming that Coriolis parameter to be proportional to the latitude. His work concerned only with the mathematical analyses of the simplified hydrodynamical equations, but it is most interesting to develop in the EPW simulations. Lindzen [14] used the β-planes approximation and Laplace's Tidal Equation to describe planetary waves in the equator and mid-latitudes. Both of these models are interesting to be discussed and simulated, but we must be careful before applying these results in the actual atmospheric disturbances.

The aim of this paper is to provide clear overview and analysis of EPW characters by examining horizontal velocity and geopotential perturbations. In this paper analytically wave solutions referencing to Matsuno equations will be found. The mathematical solutions of EPW will be simulated on an equatorial β-plane. Because a complete development of EPW would be rather complicated, at this time we only simulate the EPW in the free waves form and concentrate on the horizontal structure. The last section of this paper will be closed by a conclusion.

2 GOVERNING EQUATIONS

Equatorial waves are trapped near the equator, and they propagate in zonal and vertical directions. Mathematical interpretation of EPW's horizontal structure can be simulated by linearised shallow water equations for perturbations on a motionless basic state of mean depth H in β-plane. The advantage of the β-plane approximation is that it does not contribute nonlinear terms to the dynamical equations. This equatorial beta-plane approximation is only valid for scales L≪R, where R is the planetary radius.

Briefly, the β-plane approximation is a method to linearize Coriolis force (f) which can be approximated by expanding the latitudinal dependence of $'f'$ in a Taylor series about reference latitude ϕ_o, as follows:

$$\begin{aligned} f(\phi + \partial\phi_o) &= f(\partial\phi_o) + \left(\frac{df}{d\phi}\right)_{\phi=\phi_o} \partial\phi \\ &+ \left(\frac{df^2}{d\phi}\right)_{\phi=\phi_o} \partial\phi^2 + ... \end{aligned} \quad (1)$$

For the smallest value of $\partial\phi$, $\partial\phi$ = y/R and retaining only the first two terms (neglect the higher order terms) to yield:

$$f = f + \beta y \quad (2)$$

Equation (2) is called the mid-latitude β-plane approximation, β is a Rossby parameter ($\beta = 2\Omega\cos R^{-1}$) which can be also derived from $\partial f/\partial y$. For our model, initial Coriolis force (f_o) can be neglected at the equator, so $f = \beta y$ is the EPW's β-plane approximation.

As mentioned before, to interpret horizontal structure of EPW, we should derive shallow water equations for perturbations on a motionless basic state of mean depth H. Randall [5] explained that the shallow water equations are the simplest form of the equations of motion that can be used to describe the horizontal structure of an atmosphere. Shallow water equations can be derived from 3D momentum equations by ignoring the effects of friction:

$$\frac{D\vec{u}}{Dt} + f\,\hat{k} \times \vec{v} = -\frac{1}{\rho}\vec{\nabla}_p - g\hat{k} \quad (3)$$

$$\rho\left[\frac{D\vec{u}}{Dt} + f\,\hat{k} \times \vec{v}\right] = -\vec{\nabla}_p - \rho g\hat{k} \quad (4)$$

Here, ρ is the density of the air, f is Coriolis force, g is gravity force, $\vec{u}$ is the three-dimensional velocity vector, p is air pressure, and D is total derivative operator. By applying the perturbation method and assuming that fluid is incompressible and no fluid crosses the mean depth (H) or no share, we can now write the horizontal momentum equation as:

$$\rho\left[\frac{D\vec{v}_h}{Dt} + f\,\hat{k} \times \vec{v}\right] = -\rho g\vec{\nabla}_h \quad (5)$$

$$\left[\frac{D\vec{v}_h}{Dt} + f\,\hat{k} \times \vec{v}\right] = -g\vec{\nabla}_h \quad (6)$$

$\vec{v}_h$ is the two-dimensional velocity vector ($xi+vj$). For EPW case, wave movement is in hydrostatic balance. The speed component on the horizontal plane does not depend on the altitude, thus the continuity equation can be vertically integrated from surface $h_s = 0$ and free surface altitude h_f to yield:

$$\frac{\partial}{\partial t}(h_f - h_s) + \vec{\nabla}.\left[\vec{v_h}(h_f - h_s)\right] = 0 \quad (7)$$

$$\frac{\partial h_f}{\partial t} + \vec{\nabla}.\left[\vec{v_h} h\right] = 0 \quad (8)$$

Substitute the β-plane approximation into the equations 6 and 8, then we will find the non-linear shallow water equation in β-plane approximation:

$$\frac{Du}{Dt} - \beta y v + g\frac{\partial h}{\partial x} = 0 \quad (9)$$

$$\frac{Dv}{Dt} + \beta y u + g\frac{\partial h}{\partial y} = 0 \quad (10)$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \quad (11)$$

u and v are zonal wind velocity. In order to isolate some simple wave forms we need to linearize the equation 9-11. First we consider small perturbations on an initially motionless atmosphere, i.e. basic state winds $(u_o, v_o, w_o) = 0$. Then by applying the perturbation method in to equations 9, 10 and 11, we finally get the linear shallow water equations in equatorial β-plane:

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\beta vy+g\frac{\partial h}{\partial x} &= 0\\ \Leftrightarrow \frac{\partial}{\partial t}(\bar{u}+u')+(\bar{u}+u')\frac{\partial}{\partial x}(\bar{u}+u')&\\ +v'\frac{\partial}{\partial y}(\bar{u}+u')-\beta yv'+g\frac{\partial}{\partial x}(H+h') &= 0\\ \Leftrightarrow \frac{\partial u'}{\partial t}+\bar{u}\frac{\partial u'}{\partial x}-\beta yv'+g\frac{\partial h'}{\partial x} &= 0\\ \Leftrightarrow \frac{\partial u'}{\partial t}-\beta yv'+\frac{\partial \Phi'}{\partial x}=0 & \quad (12)\end{aligned}$$

$$\begin{aligned}\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\beta uy+g\frac{\partial h}{\partial y} &= 0\\ \Leftrightarrow \frac{\partial v'}{\partial t}+(\bar{u}+u')\frac{\partial v'}{\partial x}+v'\frac{\partial v'}{\partial y}&\\ +\beta y(\bar{u}+u')+g\frac{\partial}{\partial y}(H+h') &= 0\\ \Leftrightarrow \frac{\partial v'}{\partial t}+\bar{u}\frac{\partial v'}{\partial x}+\beta y\bar{u}+\beta yu'+g\frac{\partial h'}{\partial y} &= 0\\ \Leftrightarrow \frac{\partial v'}{\partial t}+\beta yu'+\frac{\partial \Phi'}{\partial y}=0 & \quad (13)\end{aligned}$$

$$\begin{aligned}&\frac{\partial}{\partial t}(H+h')+\frac{\partial}{\partial x}\left[(\bar{u}+u')(H+h')\right]\\ &\quad +\frac{\partial}{\partial y}\left[v'(H+h')\right]=0\\ &\Leftrightarrow \frac{\partial h'}{\partial t}+\bar{u}\frac{\partial h'}{\partial x}+H\frac{\partial u'}{\partial x}+H\frac{\partial v'}{\partial y}=0\\ &\Leftrightarrow \frac{\partial \Phi'}{\partial t}+gH\left[\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right]=0 \quad (14)\end{aligned}$$

Here, u', v', and h' are perturbation of mean zonal flow, meridional flow, and geopotential height respectively. So, linearised shallow water equations in β-plane approximation from equations 12, 13, and 14 may be briefly written as follows:

$$\frac{\partial u'}{\partial t}-\beta yv'+\frac{\partial \Phi'}{\partial x} = 0 \quad (15)$$

$$\frac{\partial v'}{\partial t}+\beta yu'+\frac{\partial \Phi'}{\partial y} = 0 \quad (16)$$

$$\frac{\partial \Phi'}{\partial t}+gH\left[\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right] = 0 \quad (17)$$

The above equations are useful to determine the mathematical structure of EPW.

2.1 Mathematical Solution of EPW in the Horizontal Structure

To simplify the determination of the EPW solution, the equation in part 15-17 must be transformed into a nondim unit form [16, 17]. This transformation is done by defining the characteristic of length unit [L], and time unit [T] in the dimensionless variables:

$$x^*=\frac{x}{[L]}\ ,y^*=\frac{y}{[L]}\ ,u^*=\frac{u}{\sqrt{gH}}\ ,v^*=\frac{v}{\sqrt{gH}}$$

$$t^*=\frac{t}{[T]}\ ,\Phi^*=\frac{\Phi}{gH}\ ,[L]=[V]\,[T]=[T]\sqrt{gH}$$

Here, x*, y*, u*, and v* are nondim units of longitude, latitude, zonal wind and meridional wind, respectively. Substituting the above nondim units into equation 12:

$$\frac{\partial\left(u^*\sqrt{gH}\right)}{\partial\left(t^*[T]\right)}-\beta y*[L]\,v^*\sqrt{gH}+\frac{\partial\left(\Phi^* gH\right)}{\partial\left(x^*[L]\right)}=0$$

$$\frac{\partial u^*}{\partial t^*}-\beta\frac{[L]^2}{\sqrt{gH}}y^*v^*+\frac{\partial \Phi}{\partial x^*}=0$$

where,

$$\beta\frac{[L]^2}{\sqrt{gH}}=1$$

Then, by using the above information, nondimensional forms of u, v, t, and Φ can be written as:

$$[T]=\left(\frac{1}{\beta\sqrt{gH}}\right)^{1/2}=\left(\frac{1}{\beta c}\right)^{1/2}$$

$$[L]=\left(\frac{\sqrt{gH}}{\beta}\right)^{1/2}=\left(\frac{c}{\beta}\right)^{1/2}$$

$$u^*=\frac{u'}{\sqrt{gH}}\ ,v^*=\frac{v'}{\sqrt{gH}}$$

$$t^*=t\,(c\beta)^{1/2}\ ,\Phi^*=\frac{\Phi'}{gH}$$

Here $c=\sqrt{gH}$ is the velocity of pure gravity waves. By taking these units, the equations (l5-17) are transformed into non-dimensional β-plane forms:

$$\begin{aligned}&\gtrdot (c\beta)^{1/2}\frac{\partial(u^*c)}{\partial t^*}-\beta y^*\left(\frac{c}{\beta}\right)^{1/2}v^*c\\ &\quad +\left(\frac{\beta}{c}\right)^{1/2}\frac{\partial\left(\Phi^* c^2\right)}{\partial x^*}=0\\ &\Leftrightarrow \beta^{1/2}c^{3/2}\frac{\partial u^*}{\partial t^*}-\beta^{1/2}c^{3/2}y^*v^*\\ &\quad +\beta^{1/2}c^{3/2}\frac{\partial \Phi^*}{\partial x^*}=0\\ &\Leftrightarrow \frac{\partial u^*}{\partial t^*}-y^*v^*+\frac{\partial \Phi^*}{\partial x^*}=0 \quad (18)\end{aligned}$$

$$\geqslant (c\beta)^{1/2} \frac{\partial (v^* c)}{\partial t^*} + \beta y^* \left(\frac{c}{\beta}\right)^{1/2} u^* c + \left(\frac{\beta}{c}\right)^{1/2} \frac{\partial (\Phi^* c^2)}{\partial y^*} = 0$$

$$\Leftrightarrow \beta^{1/2} c^{3/2} \frac{\partial v^*}{\partial t^*} + \beta^{1/2} c^{3/2} y^* u^* + \beta^{1/2} c^{3/2} \frac{\partial \Phi^*}{\partial y^*} = 0$$

$$\Leftrightarrow \frac{\partial v^*}{\partial t^*} + y^* u^* + \frac{\partial \Phi^*}{\partial y^*} = 0 \quad (19)$$

$$\geqslant (c\beta)^{1/2} \frac{\partial (\Phi^* c^2)}{\partial t^*} + c^2 \left[\left(\frac{\beta}{c}\right)^{1/2} \frac{\partial (u^* c)}{\partial x^*} + \left(\frac{\beta}{c}\right)^{1/2} \frac{\partial (v^* c)}{\partial y^*} \right] = 0$$

$$\Leftrightarrow \beta^{1/2} c^{5/2} \frac{\partial \Phi^*}{\partial t^*} + \beta^{1/2} c^{5/2} \left[\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \right] = 0$$

$$\Leftrightarrow \frac{\partial \Phi^*}{\partial t^*} + \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (20)$$

or in general, the non-dimensional linearised shallow water equations in equatorial β-plane forms may be written as follows:

$$\frac{\partial u}{\partial t} - yv + \frac{\partial \Phi}{\partial x} = 0 \quad (21)$$

$$\frac{\partial v}{\partial t} + yu + \frac{\partial \Phi}{\partial y} = 0 \quad (22)$$

$$\frac{\partial \Phi}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (23)$$

The EPW mathematical solution can be represented in the form of decaying waves in the meridional direction. Assuming waves act in the y-direction for the perturbations and propagate in the east-west direction [4, 9]:

$$\begin{bmatrix} u' \\ v' \\ \Phi' \end{bmatrix} = \begin{bmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \end{bmatrix} \exp\left[i(kx - \omega t)\right] \quad (24)$$

Here, k is zonal wavenumber and ω is frequency. The above equation comes from the Fourier component assuming that wave has a regularity in space and time so it is possible to be represented in the sinusoidal Fourier forms. If we substitute the equation (24) into equation (21-23), we will find the first-order ordinary differential equations in the y direction for meridional structure of amplitude perturbations $\hat{u}$, $\hat{v}$ and, $\hat{\Phi}$ as follows:

$$-i\omega\hat{u} - y\hat{v} + ik\hat{\Phi} = 0 \quad (25)$$

$$-i\omega\hat{v} + y\hat{u} + \frac{d\hat{\Phi}}{dy} = 0 \quad (26)$$

$$-i\omega\hat{\Phi} + ik\hat{u} + \frac{d\hat{v}}{dy} = 0 \quad (27)$$

Find the solutions for $\hat{u}$, $\hat{v}$ and, $\hat{\Phi}$ by applying some elimination techniques:

$$\begin{aligned} -i\omega\hat{u} - y\hat{v} + ik\hat{\Phi} &= 0 \\ -i\omega k\hat{\Phi} + ik^2\hat{u} + k\frac{d\hat{v}}{dy} &= 0 \\ \hline \hat{u} = \frac{1}{i(k^2 - \omega^2)} &\left[\omega y\hat{v} - k\frac{d\hat{v}}{dy}\right] \end{aligned} \quad (28)$$

and,

$$\begin{aligned} -i\omega k\hat{u} - k\hat{y}v + ik^2\hat{\Phi} &= 0 \\ -i\omega^2\hat{\Phi} + i\omega k\hat{u} + \omega\frac{d\hat{v}}{dy} &= 0 \\ \hline \hat{\Phi} = \frac{1}{i(k^2 - \omega^2)} &\left[ky\hat{v} - \omega\frac{d\hat{v}}{dy}\right] \end{aligned} \quad (29)$$

Substitute the equation (28) and (29) into equation (22) so we will find second-order ordinary differential equation for $\hat{v}(y)$:

$$\frac{d^2\hat{v}}{dy^2} + \left(\omega^2 - k^2 - \frac{k}{\omega} - y^2\right)\hat{v} = 0 \quad (30)$$

or in dimensional form, equation 30 can be written as:

$$\frac{d^2\hat{v}}{dy^2} + \left[\left(\frac{\omega^2}{gH} - k^2 - \frac{\beta k}{\omega}\right)\frac{\beta^2 y^2}{gH}\right]\hat{v} = 0 \quad (31)$$

The above equations can be simplified into first-order differential equations in order to solve depending solution of $\hat{v}$ to y:

$$\frac{d^2\hat{v}}{dy^2} - y^2\hat{v} = -C\hat{v} \quad (32)$$

Because the equatorial β-plane approximation is not valid beyond $\pm 30^o$ away from the equator we have to confine the solutions close to the equator if they are to be good approximations of the exact solutions, thus the boundary condition for this equation is:

$$\hat{v} \to 0,\ if\ y \to \pm \propto$$

C is $(\omega^2 - k^2 - \frac{k}{\omega})$, also known as dispersion relationship of EPW, thus we can modify the equation 32 in order to find the recursive relationship:

$$(D - y)(D + y)\hat{v}_C = -(C - 1)\hat{v}_C \quad (33)$$

$$(D + y)(D - y)\hat{v}_C = -(C + 1)\hat{v}_C \quad (34)$$

D is a differential operator (d/dy), then multiply equation 33 by $(D+y)$ and equation 34 by $(D-y)$:

$$(D+y)(D-y)[(D+y)\hat{v}_C] = -(C-1)[(D+y)\hat{v}_C] \quad (35)$$

$$(D-y)(D+y)[(D-y)\hat{v}_C] = -(C+1)[(D-y)\hat{v}_C] \quad (36)$$

If equation 36 is compared to equation 33 we will find a conclusion that both of these equations will have the same form if index of C in equation 33 replaced by $C+2$.

$$(D-y)(D+y)\hat{v}_{C+2} = -(C+1)\hat{v}_{C+2} \quad (37)$$

And finally the recursive relationship is:

$$\hat{v}_{C+2} = -(D-y)\hat{v}_C \quad (38)$$

Use this recursive equation in equation 34:

$$(D+y)\hat{v}_{C+2} = -(C+1)\hat{v}_C \quad (39)$$

Equation 39 shows that the solution of $\hat{v}$ is depending on C index. For $C=-1$, the solution of $\hat{v}$ is:

$$\begin{aligned}\left(\frac{d}{dy}+y\right)\hat{v}_1 &= 0\\ \hat{v}_1 &= e^{-\frac{1}{2}y^2} \quad (40)\end{aligned}$$

Then we should try to determine the other values of $\hat{v}$ by using the recursive relation and equation 40:

$$\begin{matrix}\hat{v}_2 = \left(\frac{d}{dy}-y\right)\hat{v}_0, \\ \hat{v}_4 = \left(\frac{d}{dy}-y\right)^2\hat{v}_0, \\ \hat{v}_6 = \left(\frac{d}{dy}-y\right)^3\hat{v}_0, \\ \hat{v}_8 = \left(\frac{d}{dy}-y\right)^4\hat{v}_0,\end{matrix} \left\{\begin{matrix}\hat{v}_3 = \left(\frac{d}{dy}-y\right)\hat{v}_1 \\ \hat{v}_5 = \left(\frac{d}{dy}-y\right)^2\hat{v}_1 \\ \hat{v}_7 = \left(\frac{d}{dy}-y\right)^3\hat{v}_1 \\ \hat{v}_9 = \left(\frac{d}{dy}-y\right)^4\hat{v}_1\end{matrix}\right\}$$

from these solutions, mathematically $\hat{v}$ has the value only for odd's index or $(2n+1)$, or briefly can be written as follows:

$$\hat{v}_{2n+1} = \left(\frac{d}{dy}-y\right)^n e^{-\frac{1}{2}y^2} \quad (41)$$

Remember that C from equation 41 is $2n+1$, where $n = 0,1,2,3..$, thus the equation 32 can be written in a new form:

$$\frac{d^2\hat{v}}{dy^2} - y^2\hat{v} = -(2n+1)\hat{v} \quad (42)$$

which means that the differential form in equation 42 has a solution if:

$$\omega^2 - k^2 - \frac{k}{\omega} = 2n+1 \quad (43)$$

or in the dimensional form:

$$\left[\left(\frac{\omega^2}{gH} - k^2 - \frac{\beta k}{\omega}\right)\frac{\sqrt{gH}}{\beta}\right] = 2n+1 \quad (44)$$

where n= -1,0,1,2,3.., depending solution of $\hat{v}$ to y in equation 41 can be simplified as follows:

$$\begin{aligned}\left(\frac{d}{dy}-y\right)\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d}{dy}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right)\\ &= f_1(y) \quad (45)\\ \left(\frac{d}{dy}-y\right)^2\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d^2}{dy^2}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right)\\ &= f_2(y) \quad (46)\\ \left(\frac{d}{dy}-y\right)^3\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d^3}{dy^3}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right)\\ &= f_3(y) \quad (47)\\ \left(\frac{d}{dy}-y\right)^4\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d^4}{dy^4}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right)\\ &= f_4(y) \quad (48)\\ \left(\frac{d}{dy}-y\right)^5\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d^5}{dy^5}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right)\\ &= f_5(y) \quad (49)\\ &*\\ &*\\ \left(\frac{d}{dy}-y\right)^n\hat{v}_1 &= e^{\frac{1}{2}y^2}\frac{d^n}{dy^n}\left(e^{-\frac{1}{2}y^2}\hat{v}_1\right) \quad (50)\end{aligned}$$

by using the equation 40, the equation 41 can be reduced to:

$$\hat{v}_n = e^{\frac{1}{2}y^2}\frac{d^n}{dy^n}\left(e^{-y^2}\right) \quad (51)$$

$2n+1$'s index on $\hat{v}$ can be replaced by n $(n = 0,1,2,3..)$ to have a uniform notation as a differential order. The amplitude of the wave is a positive number and then the equation (51) can be written:

$$\hat{v}_n = (-1)^n e^{\frac{1}{2}y^2}\frac{d^n}{dy^n}\left(e^{-y^2}\right) \quad (52)$$

The above equation (52) can be modified as follows:

$$(-1)^n e^{\frac{1}{2}y^2}\hat{v}_n = (-1)^n e^{y^2}\frac{d^n}{dy^n}\left(e^{-y^2}\right) \quad (53)$$

The right side of last equation is the Hermite Polynomial functions of order n, which satisfy:

$$H_n(y) = (-1)^n e^{y^2}\frac{d^n}{dy^n}\left(e^{-y^2}\right) \quad (54)$$

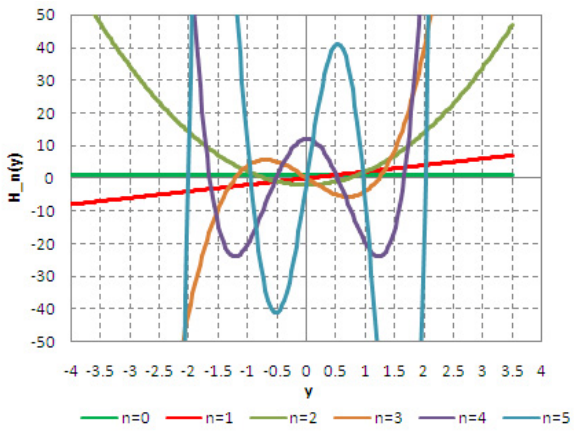

FIGURE 1: Hermite polynomial (Hn) of order n.

If it's evaluated for each n from equation (54), few of these polynomials have the values as follows:

$$H_n(y) \Leftrightarrow \begin{array}{c} H_0(y) = 1 \\ H_1(y) = 2y \\ H_2(y) = 4y^2 - 2 \\ H_3(y) = 8y^3 - 12y \\ H_4(y) = 16y^4 - 48y^2 + 12 \\ H_5(y) = 32y^5 - 160y^3 - +120y \end{array}$$

By using Hermite notation (54), then our equation set becomes:

$$\hat{v}_n = e^{-\frac{1}{2}y^2} H_n(y) \tag{55}$$

Hn (y) strongly determines the distribution of the meridional component of horizontal wind and geopotential field perturbation in the EPW problems. According to Setiawan [16], order-n in EPW is independent (orthogonal) to the other n orders, it's because the Hermite polynomials functions are orthogonal with respect to e^{-y^2}:

for $n \neq m$:

$$\int_{-\infty}^{+\infty} e^{-y^2} H_n(y) H_m(y)\, dy = 0 \tag{56}$$

for n=m :

$$\int_{-\infty}^{+\infty} e^{-y^2} H_n(y) H_m(y)\, dy = 2^n.n!\sqrt{\pi} \tag{57}$$

or,

$$\int_{-\infty}^{+\infty} e^{-y^2} H_n(y) H_m(y)\, dy = 2^n.n!\sqrt{\pi}.\delta_{mn} \tag{58}$$

because $H_n = (-1)^n e^{\frac{1}{2}y^2} \hat{v}_n$, so :

$$\int_{-\infty}^{+\infty} \hat{v}_n(y)\, \hat{v}_m(y)\, dy = 2^n.n!\sqrt{\pi}.\delta_{mn} \tag{59}$$

n and m are index of Hermite polynomials let say first and second index, δ_{mn} is Kronecker delta for positive integers m and n ($\delta_{mn} = 0$ if $m \neq n$, and $\delta_{mn} = 1$ if $m = n$). Order-n shows how the wave moving system works in a medium, thus n corresponds to the number of nodes in the meridional velocity profile in the domain $|y| < \infty$ or it's also called the mode of EPW. Next is determining the set of solutions for the horizontal structure of EPW (non-dimensional). Differentiated Hn (y) in equation (54) to y and using Leibniz notation rules in simplifying the equation:

$$\begin{aligned} \frac{d}{dy} H_n(y) &= (-1)^n 2ye^{y^2} \frac{d^n}{dy^n}\left(e^{-y^2}\right) \\ &- (-1)^n 2ye^{y^2} ye^{-y^2} \frac{d^n}{dy^n} e^{-y^2} \\ &- (-1)^n 2e^{y^2} n \frac{d^{n-1}}{dy^{n-1}} e^{-y^2} \end{aligned} \tag{60}$$

We will find some solutions of Hermite's differential which can be used to solve horizontal structure equations of EPW:

$$\frac{d}{dy} H_n(y) = 2n(-1)^{n-1} e^{y^2} \frac{d^{n-1}}{dy^{n-1}} e^{-y^2}$$

$$\frac{d}{dy} H_n(y) = 2nH_{n-1}(y) \tag{61}$$

$$\frac{d^2}{dy^2} H_n(y) = 4n(n-1) H_{n-2}(y) \tag{62}$$

Using equation (61) into the equation (28) and (29), we will find other solution forms of EPW's amplitude in the zonal and geopotential component :

$$\begin{aligned} \hat{u}_n &= \frac{1}{i(k^2-\omega^2)}\left(\omega y \hat{v}_n - k\frac{d\hat{v}_n}{dy}\right) \\ &= \frac{1}{i(k^2-\omega^2)}\left[\omega y e^{-\frac{1}{2}y^2} H_n - k\frac{d}{dy}\left(e^{-\frac{1}{2}y^2} H_n\right)\right] \\ &= \frac{1}{i(k^2-\omega^2)}\left[(\omega+k) yH_n - 2nkH_{n-1}\right] e^{-\frac{1}{2}y^2} \end{aligned} \tag{63}$$

$$\begin{aligned} \hat{\Phi}_n &= \frac{1}{i(k^2-\omega^2)}\left(ky\hat{v}_n - \omega\frac{d\hat{v}_n}{dy}\right) \\ &= \frac{1}{i(k^2-\omega^2)}\left[kye^{-\frac{1}{2}y^2} H_n - \omega\frac{d}{dy}\left(e^{-\frac{1}{2}y^2} H_n\right)\right] \\ &= \frac{1}{i(k^2-\omega^2)}\left[(\omega+k) yH_n - 2n\omega H_{n-1}\right] e^{-\frac{1}{2}y^2} \end{aligned} \tag{64}$$

with n= 0, 1, 2, 3,..., and $\omega \neq \pm k$. By inserting the value of each amplitude ($\hat{u}$, $\hat{v}$, and $\hat{\Phi}$) into the equation (24) and taking the real part, we finally find the general equations of EPW in horizontal structure (nondim) as follows:

$$u_n(x,y,t) = \frac{1}{(k^2-\omega^2)}\left[(\omega+k)\,yH_n - 2nkH_{n-1}\right] \times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (65)$$

$$v_n(x,y,t) = e^{-\frac{1}{2}y^2}H_n\cos(kx-\omega t) \quad (66)$$

$$\Phi_n(x,y,t) = \frac{1}{(k^2-\omega^2)}\left[(\omega+k)\,yH_n - 2n\omega H_{n-1}\right] \times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (67)$$

with n= 0, 1, 2, 3,..., and $\omega \neq \pm k$.Thus, solutions of EPW are propagations of the system formed by the components u, v, and Φ.

2.2 Frequency Boundary of EPW

Taking the relation dispersion in equation 43 as a quadratic equation in k:

$$k^2 + \frac{k}{\omega} + \left[2n+1-\omega^2\right] = 0 \quad (68)$$

The above equation has various real roots of k, which means that the discriminant of equation 68 must be positive:

$$4\omega^4 + 4(2n+1)\,\omega^2 + 1 > 0 \quad (69)$$

Thus, the solutions of frequency boundary in non-dimensional forms are:

$$|\,\omega\,| > \sqrt{\frac{1}{2}(2n+1) + \frac{1}{2}\sqrt{(2n+1)^2-1}} \quad (70)$$

or,

$$|\,\omega\,| < \sqrt{\frac{1}{2}(2n+1) - \frac{1}{2}\sqrt{(2n+1)^2-1}} \quad (71)$$

and in dimensional forms can be written as:

$$|\,\omega\,| > \sqrt{\left(\frac{1}{2}(2n+1) + \frac{1}{2}\sqrt{(2n+1)^2-1}\right)\beta\sqrt{gH}} \quad (72)$$

or,

$$|\,\omega\,| < \sqrt{\left(\frac{1}{2}(2n+1) - \frac{1}{2}\sqrt{(2n+1)^2-1}\right)\beta\sqrt{gH}} \quad (73)$$

These solutions provide a frequency boundary for the existence of EPW.

2.3 Yanai Wave

The case $n = 0$ is a special case corresponding to the mixed Rossby-gravity or Yanai waves. For mode case $n = 0$ the general equations for horizontal structure of Yanai wave (nondim) can be produced by using equation 65-67 as follows:

$$u'_n(x,y,t) = \frac{y}{k-\omega}e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (74)$$

$$\Phi'_n(x,y,t) = \frac{y}{k-\omega}e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (75)$$

$$v'_n(x,y,t) = e^{-\frac{1}{2}y^2}\cos(kx-\omega t) \quad (76)$$

If n=0, the dispersion relationship of EPW becomes:

$$\left(\omega^2 - k^2 - \frac{k}{\omega}\right) = 2n+1$$

$$\left(\omega^2 - k^2 - \frac{k}{\omega}\right) = 1 \quad (77)$$

then, this dispersion relationship (77) can be factored:

$$(\omega+k)\left(\omega^2 - k\omega - 1\right) = 0 \quad (78)$$

Equation 78 is a cubic equation in ω, so we will have three roots for ω when n and k are specified, namely:

$$\omega_1 = -k \quad (79)$$

$$\omega_2 = \frac{k}{2} + \sqrt{\frac{k^2}{4}+1}\,,\, k > 0 \quad (80)$$

$$\omega_3 = \frac{k}{2} - \sqrt{\frac{k^2}{4}+1}\,,\, k > 0 \; and \, k \pm \frac{1}{2}\sqrt{2} \quad (81)$$

Equation 80 is corresponding to eastward moving Poincaré wave and equation 81 can be separated into dimensional form as follows:

$$\omega_3 = \frac{k}{2}\sqrt{gH} - \frac{k}{2}\sqrt{gH}\left(1 + \frac{4\beta}{k^2\sqrt{gH}}\right)^{\frac{1}{2}} \quad (82)$$

$$\omega_3 = \begin{cases} -\frac{\beta}{k} & \text{if large } k; \\ -\left(\beta\sqrt{gH}\right)^{\frac{1}{2}} & \text{if } k \to 0. \end{cases} \quad (83)$$

From the equation 83 we can conclude that westward moving of EPW by mode $n = 0$ has mixed character. When k is large, they behave like westward Poincaré waves and when k is small or $k \to 0$, they behave like westward Rossby waves. Thus these waves are called as mixed Rossby-gravity waves or commonly known as Yanai waves. The phase speed of this wave in dimensional form can be written as:

$$c_3 = \frac{\omega_3}{k} = -\frac{\beta}{k^2}\left[\frac{1}{2} + \frac{1}{2}\left(1 + \frac{4\beta}{k^2\sqrt{gH}}\right)^{\frac{1}{2}}\right]^{-1} \quad (84)$$

Yanai waves are westward propagating ($\omega/k < 0$) relative to the mean zonal flow and dispersive (from figure 2).

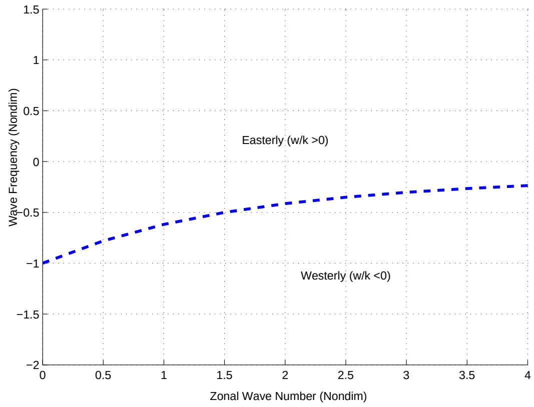

FIGURE 2: Dispersion curve of westward moving Yanai waves.

2.4 Kelvin Wave

Kelvin waves are a special case when the meridional velocity vanishes everywhere identically ($v = 0$) and equations 25-27 reduce to:

$$-i\omega\hat{u} + ik\hat{\Phi} = 0 \quad (85)$$

$$y\hat{u} + \frac{d\hat{\Phi}}{dy} = 0 \quad (86)$$

$$-i\omega\hat{\Phi} + ik\hat{u} = 0 \quad (87)$$

Eliminating the geopotential component by substituting the equation 85 into equation 87, we find:

$$\omega = \pm k \quad (88)$$

or in dimensional form:

$$\omega = \pm k\sqrt{gH} \quad (89)$$

It's known as the dispersion relation for frequency ω and wavenumber k. If equation 85 is combined to equation 86 we will find a first-order ordinary differential equation for the meridional amplitude structure of $\hat{u}$:

$$\frac{d\hat{u}}{dy} \pm y\hat{u} = 0 \quad (90)$$

The solution of this equation is:

$$\hat{u}(y) = e^{\mp\frac{1}{2}y^2} \quad (91)$$

The valid solution of 89 and 90 decays for a large y only when the negative sign in the exponential function is chosen (our boundary conditions are that the solution must decay as $y \to \infty$). It is called the solution of Kelvin waves and can then be written in dimensional forms as follows:

$$\hat{u}(y) = \hat{u}_o \exp\left(-\frac{\beta y^2 k}{\omega}\right) \quad (92)$$

$$\hat{\Phi}(y) = \sqrt{gH}\,\hat{u}_o \exp\left(-\frac{\beta y^2 k}{\omega}\right) \quad (93)$$

Then phase speed can be written in dimensional form as:

$$c = \sqrt{gH} \quad (94)$$

The second solution of 89 and 90, the one that propagates westward, would grow exponentially for large y and as such is disregarded. The solution of zonal wind and geopotential field perturbations associated with Kelvin waves in nondim forms are:

$$u'_n(x, y, t) = e^{-\frac{1}{2}y^2}\cos(kx - \omega t) \quad (95)$$

$$\Phi'_n(x, y, t) = \frac{\omega}{k}e^{-\frac{1}{2}y^2}\cos(kx - \omega t) \quad (96)$$

Thus, Kelvin waves are eastward propagating ($\omega/k > 0$) relative to the mean zonal flow and non-dispersive (figure 3). They are a form of gravity waves (94).

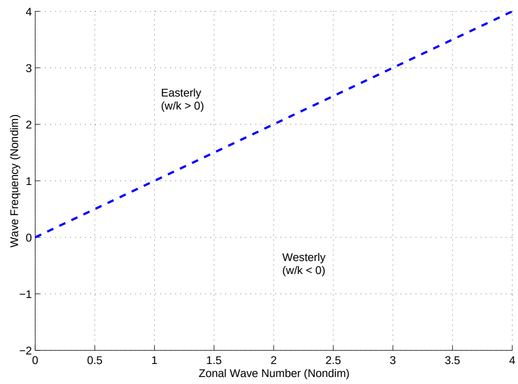

FIGURE 3: Dispersion curve of eastward moving Kelvin wave.

2.5 Equatorial Rossby and Poincaré Waves

The solutions of (30) that decay far away from the equator exist only when an important constraint connecting its coefficients is satisfied:

$$\omega^2 - k^2 - \frac{k}{\omega} = 2n + 1, \ n = 1, 2, 3, ... \quad (97)$$

The above coefficients are a dispersion relationship in ω cubic equation where the solutions associate with eastward and westward propagating waves.

For high frequencies we can neglect k/ω in 97 to obtain:

$$\omega^2 - k^2 - \frac{k}{\omega} \approx \omega^2 - k^2 = 2n+1,\ n = 1,2,3,... \quad (98)$$

and the solutions are:

$$\omega_{1,2} = \pm\sqrt{k^2 + 2n + 1} \quad (99)$$

This is a dispersion relation for equatorially trapped Poincaré waves. They propagate in either the eastward (+) or westward (-) direction. The phase speed of Poincaré waves in dimensional form is:

$$c_{1,2} = \pm c\sqrt{1 + \frac{\beta}{k^2\sqrt{gH}}(2n+1)} \quad (100)$$

Where c is pure gravity velocity and remember that meridional mode number for Poincaré waves is $n = 1, 2, 3...$ and the frequency boundary should be valid in range equation 70 or 71. These wave groups are similar to gravity-inertial waves in mid-latitudes [2].

For low frequencies we can neglect ω^2 in (97) to obtain:

$$\omega^2 - k^2 - \frac{k}{\omega} \approx -k^2 - \frac{k}{\omega} = 2n+1,\ n = 1,2,3... \quad (101)$$

and its solution is:

$$\omega_3 = -\frac{k}{k^2 + 2n + 1} \quad (102)$$

This is a dispersion relation for equatorially trapped Rossby waves. They propagate only in the westward ($\omega/k < 0$) direction. The phase speed of Rossby waves in dimensional form is:

$$c_3 = -\frac{\beta}{k^2}\left(1 + \frac{\beta}{k^2\sqrt{gH}}(2n+1)\right)^{-1} \quad (103)$$

The phase speed is inversely proportional to the square of the horizontal wavenumber. Meridional mode number for Rossby waves is $n = 1, 2, 3...$ and the frequency boundary should be valid in range equation 70 or 71. These waves are similar to their counterparts in the mid-latitudes and critically depend on the β-effect. Eastward propagation occurs when they have a high wave number [2]. Thus, both Poincaré and Rossby waves are dispersive waves. The perturbation solutions for Poincaré and Rossby waves are depending on mode-n and their frequency. In general eigenfunctions for EPW by mode $n = 1, 2, 3...$ are:

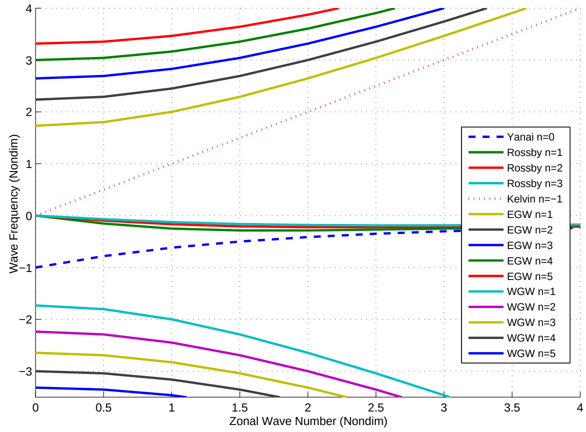

FIGURE 4: The dispersion relation for free equatorial planetary waves (EPW) in nondimensionalized wavenumber and frequency (Kelvin waves, Yanai waves, Rossby waves, East Poincaré waves (EGW) and West Poincaré waves (WGW).

for mode-n=1,

$$\begin{aligned} u'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)}\left[2y^2(\omega+k) - 2k\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (104) \\ \Phi'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)}\left[2y^2(\omega+k) - 2\omega\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (105) \\ v'_n(x,y,t) &= 2ye^{-\frac{1}{2}y^2}\cos(kx-\omega t) \quad (106) \end{aligned}$$

for mode-n=2,

$$\begin{aligned} u'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)}\left[\left(4y^3-2y\right)(\omega+k) - 8yk\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (107) \\ \Phi'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)}\left[\left(4y^3-2y\right)(\omega+k) - 8y\omega\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (108) \\ v'_n(x,y,t) &= \left(4y^2-2\right)e^{-\frac{1}{2}y^2}\cos(kx-\omega t) \quad (109) \end{aligned}$$

and for mode-n=3,

$$\begin{aligned} u'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)} \\ &\times \left[\left(8y^4-12y^2\right)(\omega+k) - \left(24y^2-12\right)k\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (110) \\ \Phi'_n(x,y,t) &= \frac{1}{(k^2-\omega^2)} \\ &\times \left[\left(8y^4-12y^2\right)(\omega+k) - \left(24y^2-12\right)\omega\right] \\ &\times e^{-\frac{1}{2}y^2}\sin(kx-\omega t) \quad (111) \\ v'_n(x,y,t) &= \left(8y^3-12y\right)e^{-\frac{1}{2}y^2}\cos(kx-\omega t) \quad (112) \end{aligned}$$

3 THEORETICAL SIMULATIONS

As we mentioned above that the theoretical simulations of equatorial planetary waves will be designed based on shallow water equations on a motionless basic state of mean depth H in β-plane. The simulations are developed in a barotropic model. The simplest condition of barotropy is given by assuming that we have a homogeneous model atmosphere. A single layer of homogeneous, incompressible fluid with a free surface has been applied and it is assumed that Coriolis parameter is proportional to the latitude.

Based on meridional amplitude simulations, in general for EPW $n = 0$ perturbation of meridional wind associated with existence of EPW has the greatest meridional wind amplitude at the equator and Gaussian decays when away from the equator. EPW by mode n=1 has greatest amplitude of meridional wind perturbation (v') at latitude $y = \pm 1$ (nondim) and v' equal to zero at the equator while EPW by mode $n = 2$, amplitude of v' equal to zero at latitude $y = \pm 1/2\sqrt{2}$ (nondim) and greatest amplitudes outside the tropics. The distributions are graphically shown in the figure 5. The waves are trapped in meridional direction (figure 5). This trapping occurs symmetrically north and south of the equator, so that the equatorial region becomes a waveguide. The further simulations of EPW will be discussed below in the special modes of free waves such as Yanai waves, Kelvin waves, Poincaré waves and even Rossby waves.

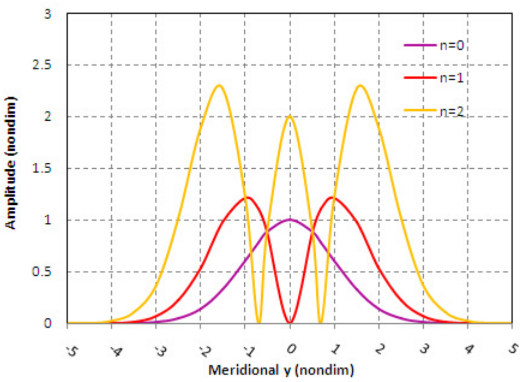

FIGURE 5: Experiment of meridional wind amplitude associated with EPW in the north-south direction for mode n = 0, 1 and 2.

3.1 Yanai and Kelvin Waves Simulations

The peak of meridional wind perturbations of Yanai waves will occur at the equator (Gaussian distribution centered at the equator) and decays moving away from the equator, while zonal wind perturbations have their greatest amplitude at latitude $y = \pm 1$ (nondim) and zero around the equatorial latitude belt, as well as to geopotential fields (figure 6a). Yanai waves' amplitude for zonal wind, meridional wind, and geopotential field component are identically symmetric with different forms. For this case amplitude of the meridional wind is Gaussian symmetry and amplitude of zonal wind and geopotential field are symmetric with two peaks at latitude $y = \pm 1$ (nondim).

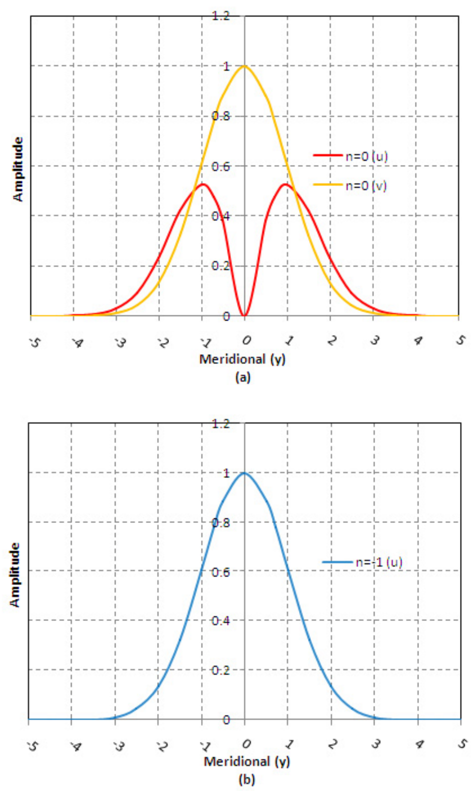

FIGURE 6: Experiment of zonal wind amplitude (red) and meridional wind amplitude (yellow) associated with Yanai waves in the north-south direction for Yanai waves mode n = 0 (a). Zonal wind amplitude (blue) associated with Kelvin waves (b) with $k = 0.5$ dan ω = -0.65326.

For Kelvin waves' case, the waves are only corresponding to symmetry zonal wind and geopotential height while perturbation of meridional wind (southerly velocity component) is identically zero at about the equator (figure 6b). The peak amplitude of zonal wind perturbation is at the equator and decays in a Gaussian form when away from the equator. The shape of zonal wind and geopotential field amplitude are symmetrical Gaussian. The character of zonal

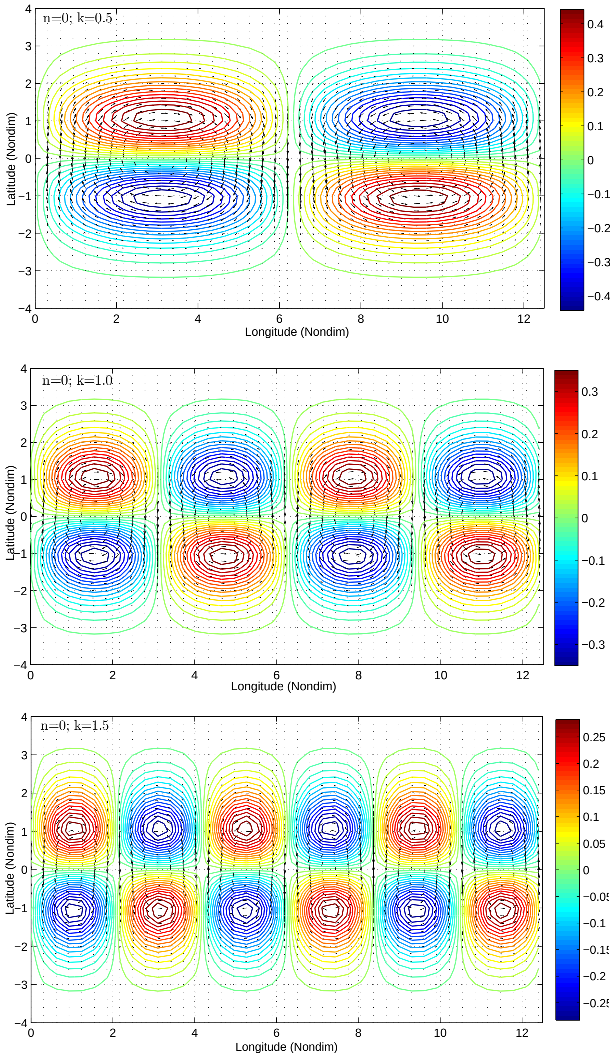

FIGURE 7: Simulation of geopotential field and horizontal wind perturbations corresponding to Yanai waves mode n= 0 with zonal wavenumber (k)=0.5,=1.0, and 1.5, respectively.

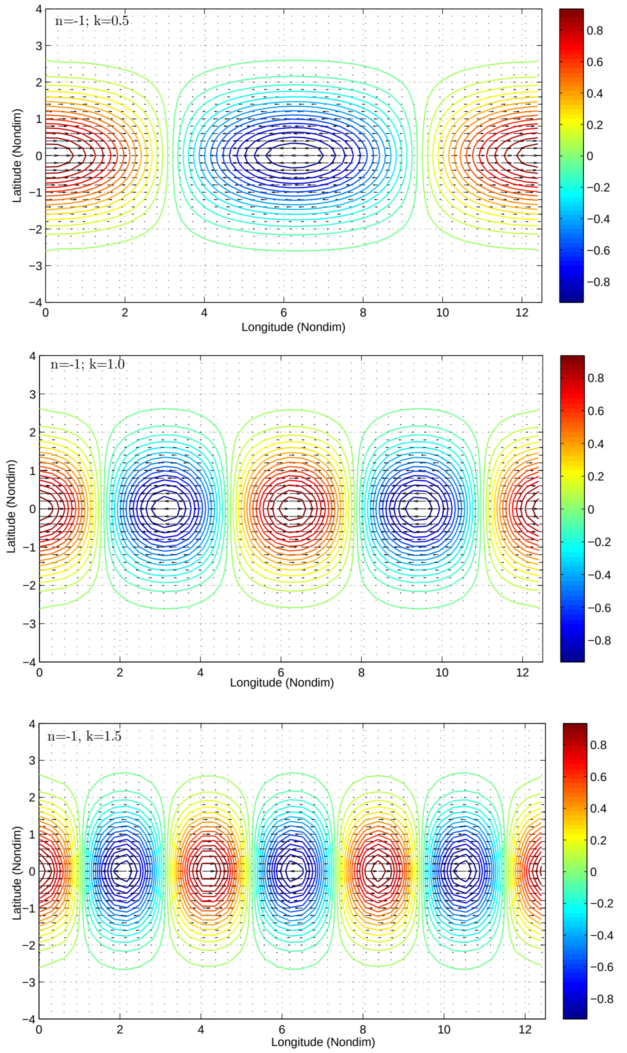

FIGURE 8: Simulation of geopotential field and horizontal wind perturbations corresponding to Kelvin waves mode n= -1 with zonal wavenumber (k)=0.5,=1.0, and 1.5, respectively.

wind perturbation in Kelvin waves is similar to meridional wind perturbation of Yanai waves.

The perturbations of three components (u', v' and Φ') associated with Yanai and Kelvin waves are simulated in figure 7. From the results of simulation, in general it is clear that the characters of Yanai or Rossby-Gravity waves have a maximum oscillation of the meridional wind perturbation at the equator and exponentially decays (Gaussian) when away from the equator. The decaying is characterized by a weakening of the meridional wind perturbations (look at the arrows shape), while the zonal wind is zero at the equator. In the higher latitudes $y > 1$ and $y < -1$ perturbation of zonal wind (u') and geopotential field (Φ') will be approximately in geostrophic balance while near the equator ageostrophic wind components predominate. The value of y domain can be written in dimensional form as follows:

$$\left(\frac{1}{\beta}\sqrt{gH}\right)^{\frac{1}{2}} < y < -\left(\frac{1}{\beta}\sqrt{gH}\right)^{\frac{1}{2}}$$

Figure 7 is Yanai waves simulation with $t = 0$, waves with larger k values will produce more pressure cells where high pressure and low pressure formed are not symmetric to the equator (regularly changes to the longitude). The oscillations of zonal wind and geopotential height are antisymmetric but the oscillations of meridional wind are symmetric. In addition, the number of cells that formed showed that a higher k value (zonal wavenumber) will produce more antisymmetric zonal waves (s), but when k increases, the wavelength (λ) will decrease. If $k = 0.5$ there are only a pair of high-pressure cell and a pair of low pressure cells ($s = 1$) that spread in a circumference of the earth's atmosphere while if $k = 1$ and $k = 1.5$ they increase to two ($s = 2$) and three times ($s = 3$) of $k = 0.5$. These waves move toward the west and do not have a constant form when they propagate (dispersive).

In the case of free Kelvin waves, Figure 8d-f describe some simulations of the horizontal perturbations of geopotential and horizontal wind velocity for Kelvin wave $n = -1$ with $k = 0.5$, 1.0, and 1.5. These provide information that the existence of Kelvin waves are characterized by disturbances of zonal winds which are stronger in the equator and decay when away from the equator, and there is no oscillation of meridional wind component. A higher k value (zonal wave number) will produce more symmetric zonal wave numbers (s) but the wavelength (λ) is small. These waves move toward the east and have a constant form when they propagate. The phase speed of these waves is easterly relative to the mean zonal flow. Typical values of the Kelvin wave speed are in range $c \approx 10$ - 50 m/s in troposphere (corresponding to $H = 10$ - 250 m) with higher values in middle atmosphere. The existence of Kelvin waves in the atmosphere will change the average of zonal wind profile in latitude (figure 9).

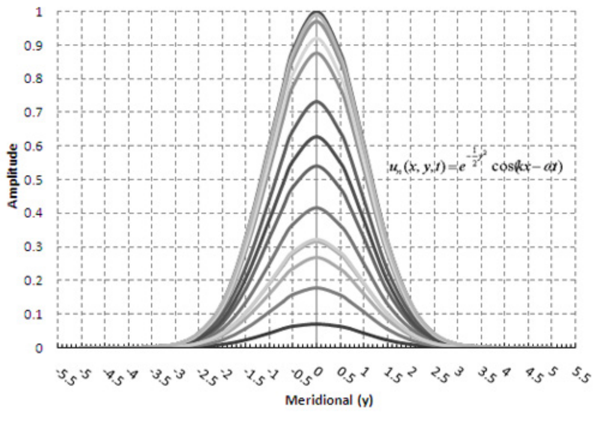

FIGURE 9: Experiment of mean zonal wind profile due to Kelvin wavedriving (nondim).

3.2 Equatorial Rosbby Waves and Poincaré Waves Simulations

Rossby waves propagate only to the west, whereas their energy (group velocity) may propagate to the east or west (dispersive). Further, they are described in figure 10. Rossby waves with eigensolution for mode $n = 1$ are characterized by the geostrophic relationship between pressure (geopotential fields) and velocity winds. The strong zonal velocity is found along the equator. Rossby waves will be in geostrophic balance between the pressure gradient and wind at domain $y > 0.5$ and $y < -0.5$ (nondim). Meridional wind perturbations vanish along the equator. Zonal wind (u') and geopotential field (Φ') perturbations are symmetric relative to the latitude but the meridional wind perturbation (v') is antisymmetric. For Rossby waves with eigensolution for mode $n = 2$ are characterized by the geostrophic relationship between pressure (surface elevation) and velocity fields at domain domain $y > 0.5$ and $y < -0.5$ (nondim). The differences from n=1 are these waves do not have wind field perturbation along the equator, the maximum peak for zonal wind perturbation is approximately at $y = \pm 1$, v' is symmetric, and both of u' and Φ' are antisymmetric relative to latitude. Rossby waves by mode $n = 3$ are similar to eigensolution of $n = 1$, but for this case the peak of u' are approximately in 3 regions namely y=-1, 0, and 1. In general, we can conclude that Rossby waves are quasi geotrosphic motions, and if we assume for a small k, the Rossby waves will be non-dispersive waves. Consequently, the phase speed of Rossby waves of different modes is $c/3, c/5, c/7$. (eq.102) or phase speed for $n = 1 > n = 2 > n = 3$. In other words, the higher-order Rossby modes are much slower and phase speed of Rossby waves $<$ Kelvin waves.

Eastward Poincaré or east propagating Inertio-gravity waves have phase and group velocities toward the east. From our simulations (figure 11), Poincaré waves by mode $n = 1$ are characterized by symmetric geopotential height and zonal wind perturbations. Meridional wind perturbations vanish along the equator and appear at domain about $y \pm 0$ (antisymmetry). These waves are also characterized by relationships between Pressure and velocity fields. For case mode n=2, Poincaré waves have symmetric meridional wind perturbations about the equator and no evidence of zonal wind perturbations. For case $n = 3$, is similar to $n = 1$, but number of the pressure cells increase to be 5 cells relative to Earth's latitude. The higher-order East propagating Poincaré modes are much faster and phase speed of these waves are more than Kelvin waves or in other words phase speed of eastward Poincaré waves $>$ Kelvin waves $>$ Rossby waves.

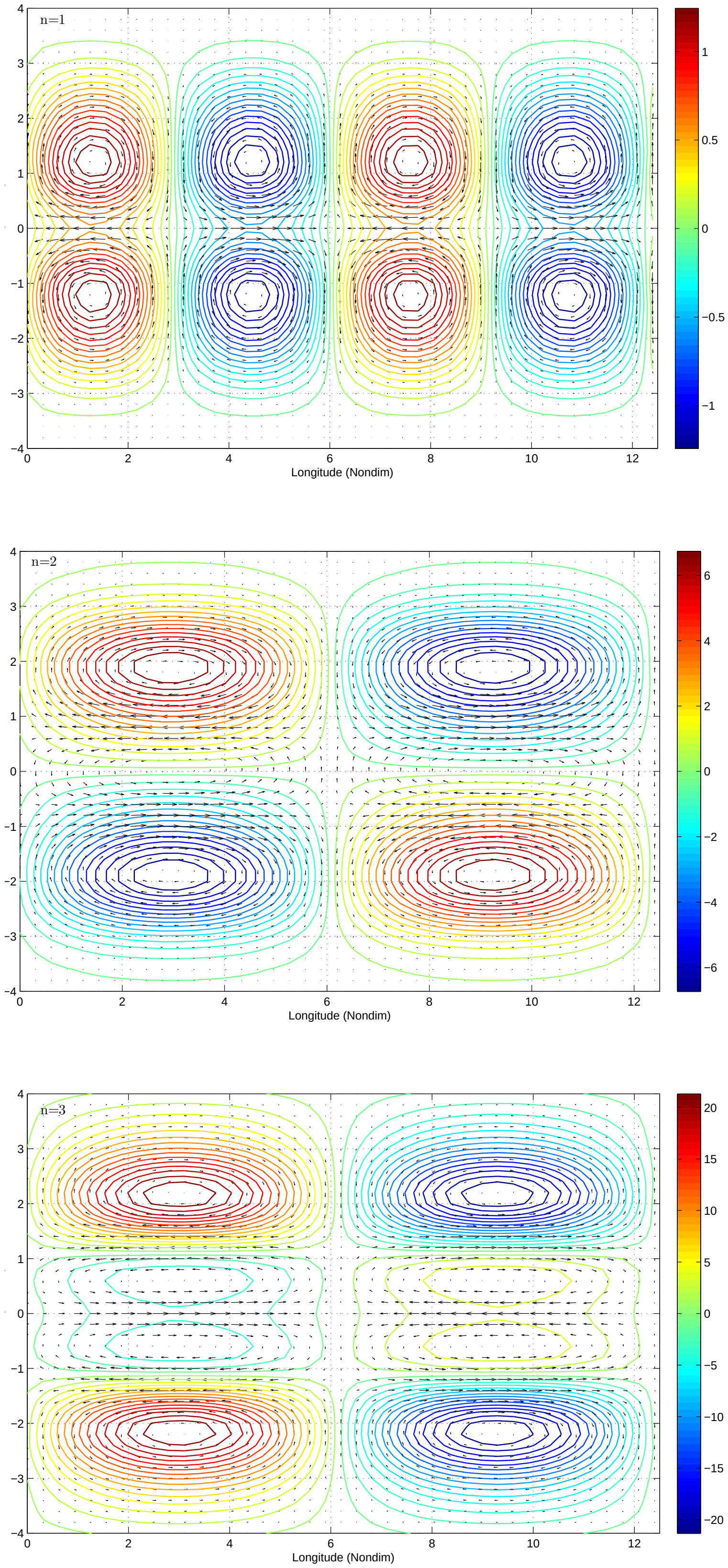

FIGURE 10: Simulation of horizontal wind and geopotential field perturbations corresponding to equatorial Rossby waves mode n= 1, 2, and 3.

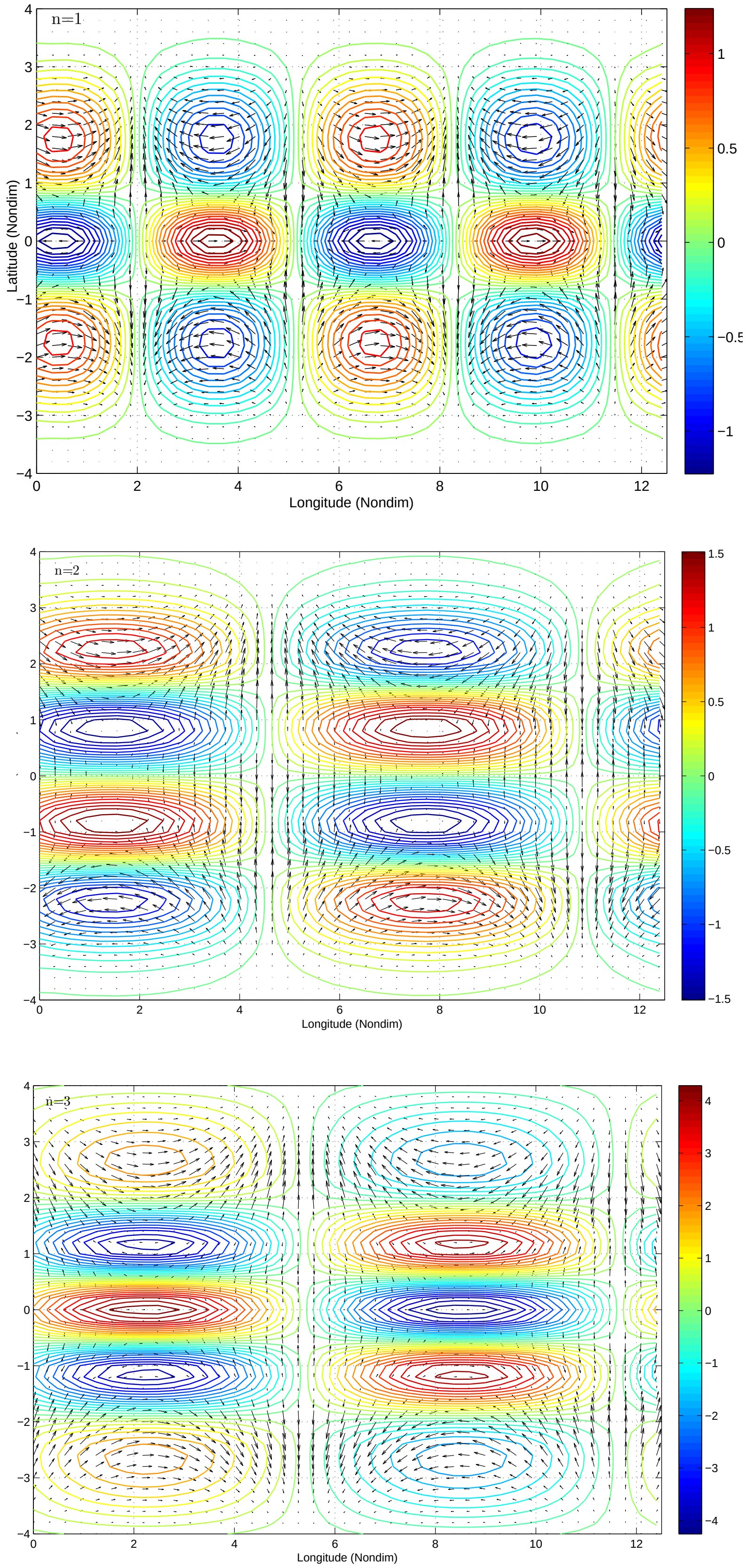

FIGURE 11: Simulation of horizontal wind and geopotential field perturbations corresponding to equatorial Eastward Poincaré waves mode n= 1, 2, and 3.

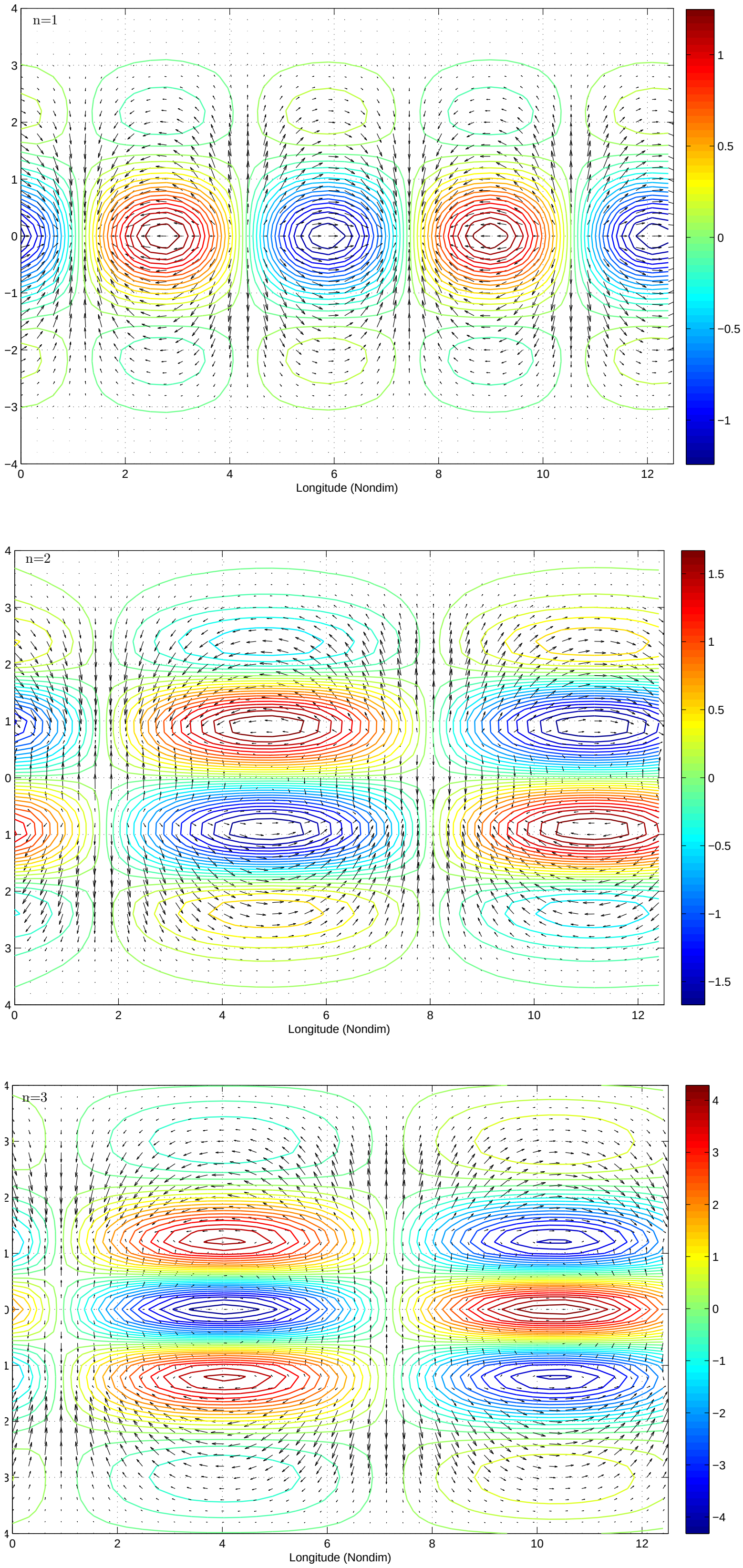

FIGURE 12: Simulation of horizontal wind and geopotential field perturbations corresponding to equatorial Westward Poincaré waves mode n= 1, 2, and 3.

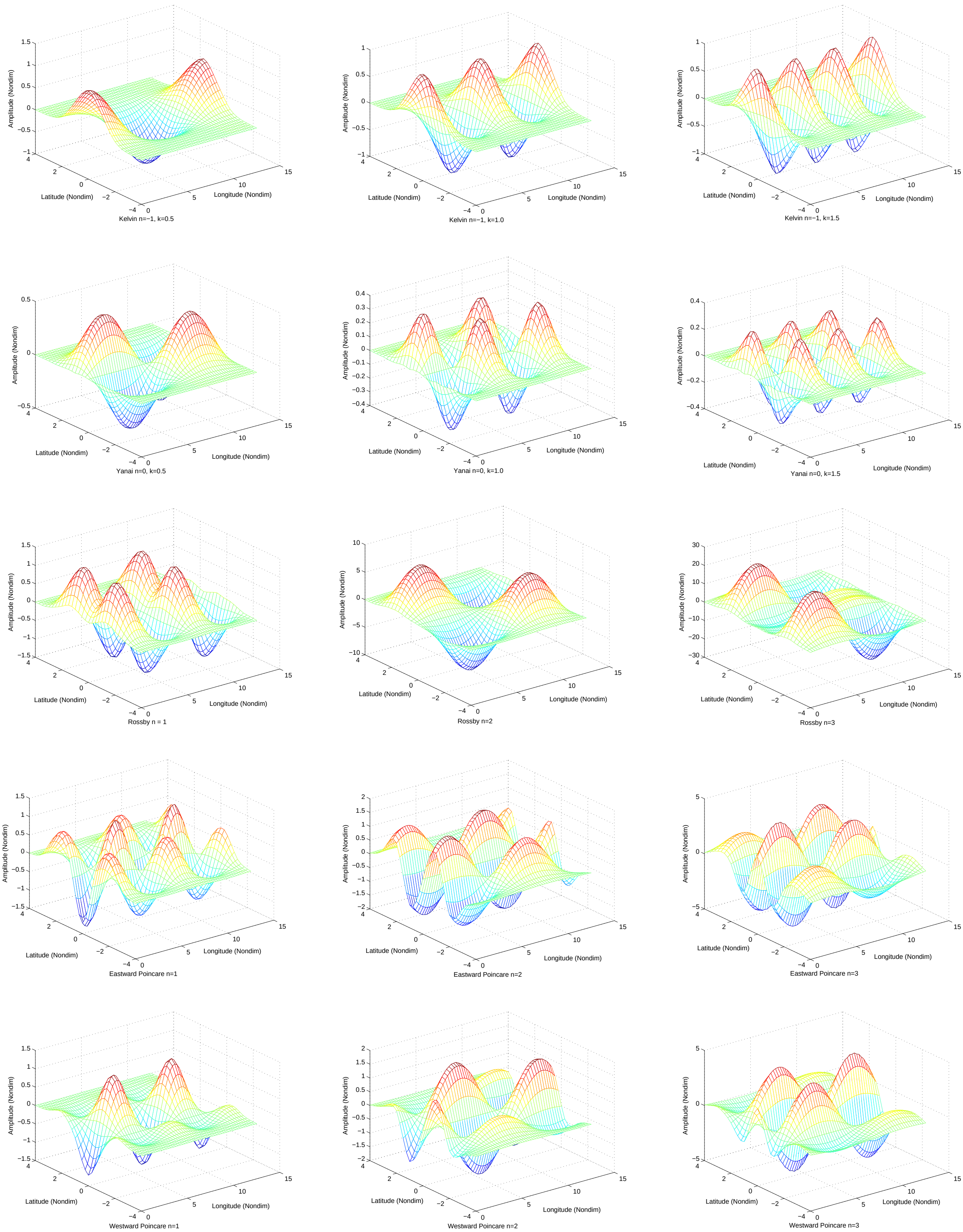

FIGURE 13: Simulation of surface elevation corresponding to Kelvin waves, Yanai waves, Rossby waves, eastward Poincaré waves and westward Poincaré waves, respectively.

Westward Poincaré waves have phase velocity toward the west and group velocity also toward the west except for very low zonal wavenumbers. Further, they are described in figure 12. West propagating Poincaré waves with eigensolution for mode n=1 are characterized by strong zonally symmetric component, as well as the meridional wind perturbation vanishes along the equator. Both zonal wind and geopotential field perturbations are symmetric but meridional wind perturbations are antisymmetric when away from the equator. Simulations of westward Poincaré waves with eigensolution mode $n = 2$ and 3 are similar to eastward Poincaré waves. The differences are in distribution of horizontal wind directions and forms of geopotential fields relative to latitude. Strong horizontal wind perturbations are near the domain $1 < y < 3$ and $-3 < y < -1$ (nondim). Simulation of surface elevation corresponding to Kelvin waves, Yanai waves, Rossby waves, eastward Poincaré waves and westward Poincaré waves are described in figure 13.

4 CONCLUSIONS

From these simulations we can conclude that k and ω are parameters which control the type of wave patterns. Eigenfunction gradually changes if there is a differences between k and ω. Further, the general solutions of EPW are formed by functions-n, so if n is an odd number then v' is an odd function and u' and Φ' are even functions with respect to y. Consequently these waves have symmetric components of u' and zero or antisymmetric components of v' relative to latitude, for example Rossby $n = 1, 3$, Kelvin $n = -1$, eastward Poincaré waves $n = 1, 3$, and westward Poincaré waves $n = 1, 3$. If n is even number, each components are reversed, for example Yanai $n = 0$, Rossby $n = 2$, eastward Poincaré waves $n = 2$ and westward Poincaré waves $n = 2$.

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